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Time : 10 Hour | STD 11 Maths | Total Marks : 200

kd90+ ch- 7 binomial theorem

*** Choose the right answer from the given options. [1 Marks Each]** **[79]**

1. In the expansion of $\left(\sqrt{2}+\sqrt[5]{3}\right)^{120}$ the number of irrational terms is:

   (A) 12 (B) 13 (C) 108 (D) 54

   Ans. :

   Solution:

   Total number of rational terms is

   $\frac{120}{\mathrm{L.C.M}(5,2)}+1$

   $\frac{120}{10}+1$

   $=13$

   Hence total number of irrational terms are

   = 121 - 13

   = 108

2. If in the expansion of $(1+\mathrm{y})^{\mathrm{n}}$, the coefficients of 5^{th}, 6^{th} and 7^{th} terms are in A.P., then n is equal to:

   (A) 7, 11 (B) 7, 14 (C) 8, 16 (D) None of these.

   Ans. :

   b. 7, 14

   Solution:

   Coefficients of the 5^{th}, 6^{th} and 7^{th} terms in the given expansion are ${}^{\mathrm{n}}\mathrm{C}_4, {}^{\mathrm{n}}\mathrm{C}_5$ and ${}^{\mathrm{n}}\mathrm{C}_6$.

   These coefficients are in AP.

   Thus, we have

   $2\,{}^{\mathrm{n}}\mathrm{C}_5={}^{\mathrm{n}}\mathrm{C}_4+{}^{\mathrm{n}}\mathrm{C}_6$

   On dividing both sides by ${}^{\mathrm{n}}\mathrm{C}_5$, we get

   $2=\frac{{}^{\mathrm{n}}\mathrm{C}_4}{{}^{\mathrm{n}}\mathrm{C}_5}+\frac{{}^{\mathrm{n}}\mathrm{C}_6}{{}^{\mathrm{n}}\mathrm{C}_5}$

   $\Rightarrow 2=\frac{5}{\mathrm{n}-4}+\frac{\mathrm{n}-5}{6}$

   $\Rightarrow 12\mathrm{n}-48=30+\mathrm{n}^2-4\mathrm{n}-5\mathrm{n}+20$

   $\Rightarrow \mathrm{n}^2-21\mathrm{n}+98=0$

   $\Rightarrow (\mathrm{n}-14)(\mathrm{n}-7)=0$

   $\Rightarrow \mathrm{n}=7,14$

3. Choose the correct answer.

   If the middle term of $\left(\frac{1}{\mathrm{x}}+\mathrm{x}\sin\mathrm{x}\right)^{10}$ is equal to $7\frac{7}{8}$, then value of x is:

(A) $2n\pi + \frac{\pi}{6}$. (B) $n\pi + \frac{\pi}{6}$. (C) $n\pi + (-1)^n \frac{\pi}{6}$. (D) $n\pi + (-1)^n \frac{\pi}{3}$.

Ans. :

c. $n\pi + (-1)^n \frac{\pi}{6}$.

Solution:

Given expression is $\left(\frac{1}{x} + x\sin x\right)^{10}$

Since, n = 10 (even), so there is only one middle term which is, 6^{th} term.

$\therefore T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{1}{x}\right)^{10-5} (x\sin x)^5$

$\Rightarrow \frac{63}{8} = {}^{10}C_5 \sin^5 x$ (given)

$\Rightarrow \frac{63}{8} = 252 \times \sin^5 x \Rightarrow \sin^5 x = \frac{1}{32} \quad \Rightarrow \sin x = \frac{1}{2} \Rightarrow \sin x = \sin \frac{\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{6}$

4. The middle term in the expansion of $\left(1 + \frac{1}{x^2}\right)\left(1 + x^2\right)^n$ is:

(A) ${}^{2n}Cn\ x^{2n}$ (B) ${}^{2n}Cn\ x^{-2n}$ (C) ${}^{2n}Cn$ (D) ${}^{2n}C_{n-1}$

Ans. :

c. ${}^{2n}Cn$

5. What is the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3} - \frac{3}{y\sqrt{x}}\right)^{12}$?

(A) C(12, 7) x^3 y^{-3} (B) C(12, 6) x^{-3} y^3 (C) C(12, 7) x^{-3} y^3 (D) C(12, 6) x^3 y^{-3}

Ans. :

d. C(12, 6) x^3 y^{-3}

6. Number of irrational terms in the expansion of $\left(5^{\frac{1}{6}} + 2^{\frac{1}{8}}\right)^{100}$ is:

(A) 96 (B) 97 (C) 98 (D) 99

Ans. :

b. 97

Solution

$T_{r+1} = {}^{100}C_r\, 5^{\frac{(r-100)}{6}} 2^{\frac{r}{8}}$

Hence we get rational terms when

r = 8k where k is an integer and $\frac{8k-100}{6}$ is an integer

r = 16, 40, 64, 88

Hence we get in total 4 rational terms.

However, total number of terms will be 101

Hence total number of irrational terms is 101 - 4

= 97 terms.

7. If tr is the rth term in the expansion of (1+ x)101, then what is the ratio $\frac{t20}{t19}$ equal to?

(A) $\frac{20x}{19}$ (B) $83x$ (C) $19x$ (D) $\frac{83x}{19}$

Ans. :

d. $\frac{83x}{19}$

8. r and n are positive integers r > 1, n > 2 and coefficient of (r+2)th term and 3rth term in the expansion of $(1 + x)^{2n}$ are equal, then n equals:

(A) 3r (B) 3r + 1 (C) 2r (D) 2r + 1

Ans. :

c. 2r

9. The coefficient of the term independent of x in the expansion of $\left(\frac{\sqrt{x}}{3}+\frac{3}{2x^2}\right)10$ is:

(A) $\frac{5}{4}$ (B) $\frac{7}{4}$ (C) $\frac{9}{4}$ (D) None of these

Ans. :

a. $\frac{5}{4}$

10. The coefficient of the middle term in the expansion of (2 + 3x)4 is:

(A) 5! (B) 6 (C) 216 (D) 8!

Ans. :

c. 216

Solution:

If the exponent of the expression is n, then the total number of terms is n + 1.

Hence, the total number of terms is 4 + 1 = 5.

Hence, the middle term is the 3rd term.

Therefore, $T_3 = {}^4C_2 \times (2)^2 \times (3x)^2$

$T_3 = (6) \times (4) \times (9x^2)$

$T_3 = 216x^2$.

Therefore, the coefficient of the middle term is 216.

11. $(\sqrt{3}+1)^5-(\sqrt{3}+1)^5=$

(A) 152 (B) 142 (C) 124 (D) 162

Ans. :

a. 152

Solution:

In the above binomial expansion, the terms at the odd position will get eliminated.

We would be left with

$2({}^5C_1(\sqrt{3})^4+{}^5C_3(\sqrt{3})^2+{}^5C_5)$

$=2(5(3^2)+10(3)+1)$

$=2(45+30+1)$

$= 2(76)$
$= 152$

12. The coefficient of x - 12 in the expansion of $\left(\frac{x+y}{x3}\right)^{20}$ is:

(A) $^{20}C_8$ (B) $^{20}C_8\ y^8$ (C) $^{20}C_{12}$ (D) $^{20}C_{12}$ y12

Ans. :

b. $^{20}C_8\ y^8$

13. The coefficient of x^{-17} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is:

(A) 1365 (B) -1365 (C) 3003 (D) -3003

Ans. :

b. -1365

Solution:

Suppose the (r + 1)th term in the given expansion contains the coefficient of x^{-17}.

Then, we have

$T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r$

$\Rightarrow (1)^r\ {}^{15}C_{r-1}\ x^{60-4r-3r}$

For this term to contain x^{-17}, we mst have

$60 - 7r = -17$

$\Rightarrow 7r = 77$

$\Rightarrow r = 11$

$\therefore$ Required coefficient $= (-1)^{11}\ {}^{15}C_{11} = -\frac{15\times14\times13\times12}{4\times3\times2} = -1365$

14. If the coefficients of 2nd, 3rd and the 4th terms in the expansion of (1 + x)n are in A.P., then value of n is:

(A) 3 (B) 7 (C) 11 (D) 14

Ans. :

b. 7

15. The sum of the coefficient in the expansion of $(x + y)^n$ is 4096. The greatest coefficient in the expansion is:

(A) 1024 (B) 924 (C) 824 (D) 724

Ans. :

b. 924

Solution:

$(x + y)^n$, Sum of coefficient = 4096

When x = y = 1, if n = 12

⇒ $(1 + 1)^{12} = 2^{12} = 4096$

⇒ Hence, greatest coefficient

${}^{n}C_{\frac{n}{2}} = {}^{12}C_6 = \frac{12!}{6!6!} = 924$

Hence, this is the answer.

16. The value of $\sum_{r=0}^{n} a_{2r-1}$ is:

(A) $9^n - 1$ (B) $9^n + 1$ (C) $9^n - 2$ (D) $9^n + 2$

Ans. :

b. $9^n + 1$

Solution:

$(1 + 4x + 4x^2)^n = a_0 + a_1x + a_2x^2 + \dots (a_{2n}x^{2n})$

Substituting x = 1 we get

$9^n = a_0 + a_1 + a_2 + \dots (a_{2n})$

Substituting x = -1 we get

$1 = a_0 - a_1 + a_2 - a3 \dots (a_{2n})$

Adding both we get

$2(a_0 + a_2 + a_4 + \dots a^{2n}) = 9^n + 1$

Hence

$\sum_{k=0}^{n} a_{2k} = 9^n + 1$

17. Number of rational terms in the expansion of $\left(\sqrt{2} + \sqrt[4]{3}\right)^{100}$ is:

(A) 25 (B) 26 (C) 27 (D) 28

Ans. :

b. 26

Solution:

The general term for the following expression is $T_{r+1} = {}^{100}C_r 2^{50-\frac{r}{2}} \cdot 3\frac{r}{4}$.

Hence we get rational terms for

r = 0, 4, 8, 12 100

$a_n = a + (n - 1).d$

100 = 0 + (n - 1).4

25 = n - 1

n = 26

18. If the coefficient of x in $\left(x^2 + \frac{\lambda}{x}\right)^5$ is 270, then $\lambda =$

(A) 3 (B) 4 (C) 5 (D) None of these.

Ans. :

a. 3

Solution:

The coefficient of x in the given expansion where x occurs at the (r + 1)th term.

We have,

$^{15}C_r(x^2)^{5-r}\left(\frac{\lambda}{x}\right)^r$

$= {}^{15}C_r\ \lambda^r\ x^{10-2r-r}$

For it to contain x, we must have

$10-3r=1$

$\Rightarrow r=3$

Coefficient of x in the given expansion,

$= {}^{15}C_3\ \lambda^3 = 10\lambda^3$

Now, we have

$10\lambda^3 = 270$

$\Rightarrow \lambda^3 = 27$

$\Rightarrow \lambda = 3$

19. The coefficient of x^8 y^{10} in the expansion of $(x+y)^{18}$ is:

(A) $^{18}C_8$ (B) $^{18}p_{10}$ (C) 2^{18} (D) None of these.

Ans. :

a. $^{18}C_8$

Solution:

Suppose (r + 1)th term in the given expansion is independent of x.

Then, we have

$T_{r+1} = {}^{18}C_r(x)^{18-r}\ y^r$

For this term to be independent of x, we must have

$r=10$

Hence, the required coefficient is $^{18}C_{10}$ or $^{18}C_8$

20. The coefficient of x^3 in $\left(\sqrt{x^5}+\frac{3}{\sqrt{x^3}}\right)^5$ is:

(A) 0 (B) 120 (C) 420 (D) 540

Ans. :

d. 540

Solution:

$r = \frac{6\times\frac{5}{2}-3}{\frac{5}{2}+\frac{3}{2}} = \frac{15-3}{4} = 3$

$\therefore$ Coefficient of x^3 is $^6C_3 3^3$

$= \frac{6\times5\times4}{3\times2\times1}\cdot 27$

= 5 × 4 × 27 = 540

21. Expand the following binomials: $(x - 3)^5$

(A) $x^5 + 25x^4 + 90x^3 - 270x^2 + 405x - 243$
(B) $x^5 - 15x^4 + 90x^3 - 270x^2 - 405x - 243$
(C) $x^5 - 15x^4 + 80x^3 - 270x^2 + 405x - 243$
(D) $x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

Ans. :

d. $x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

Solution:

$(x - 3)^5 = {}^5C_0x^5 + {}^5C_1x^4(-3)^1 + {}^5C_2x^3(-3)^2 + {}^5C_3x^2(-3)^3 + {}^5C_4x(-3)^4 + {}^5C_5(-3)^5$

$= x^5 -15x^4 + 90x^3 - 270x^2 + 405x - 243$

22. The number of terms in the expansion of $[(a + 4b)^3 (a - 4b)^3]^2$ are:

(A) 6
(B) 7
(C) 8
(D) 32

Ans. :

b. 7

Solution:

$[(a + 4b)^3 (a - 4b)^3]^2$

$= [(a + 4b)(a - 4b)]^6$

$= [a^2 - 16b^2]^6$

Hence total number of terms is n + 1
Here n = 6
Therefore, total number of terms is 7.

23. The coefficients of the expansions are arranged in an array. This array is called ....

(A) Pascal's Triangle
(B) Binomial Triangle
(C) Fibonacci Triangle
(D) Pingla Triangle

Ans. :

a. Pascal's Triangle

24. ${}^{15}C_3 + {}^{15}C_5 + ... + {}^{15}C_{15}$ will be equal to:

(A) 2^{14}
(B) $2^{14} - 15$
(C) $2^{14} + 15$
(D) $2^{14} - 1$

Ans. :

b. $2^{14} - 15$

Solution

We know

${}^{15}C_1 + {}^{15}C_3 + {}^{15}C_5 + ...{}^{15}C_{15} = 2^{15 - 1}$

$\therefore {}^{15}C3 + {}^{15}C_5 + ... + {}^{15}C_{15} = 2^{14} - 15$

25. If the 4th term in the binomial expansion of (p + 1)n is $\frac{5}{2}$ then:

(A) $n=8, p=6$ (B) $n=8, p=\frac{1}{2}$ (C) $n=6, p=\frac{1}{2}$ (D) $n=6, p=6$

Ans. :

c. $n=6, p=\frac{1}{2}$

26. The total number of terms in the expansion of $(x+a)^{100}+(x-a)^{100}$ after simplification is:

(A) 202 (B) 51 (C) 50 (D) None of these.

Ans. :

b. 51

Solution:

Here, n i.e. 100 is even.

$\therefore$ Total number of terms in the expansion $=\frac{n}{2}+1=\frac{100}{2}+1=51$

27. The number of rational terms in the expansion of $(9^{\frac{1}{4}}+8^{\frac{1}{6}})^{1000}$ is:

(A) 500 (B) 400 (C) 501 (D) None of the above

Ans. :

c. 501

Solution

The general term in the expansion of $(9^{\frac{1}{4}}+8^{\frac{1}{6}})^{1000}$ is

$T_{r+1}={}^{1000}Cr\left(9^{\frac{1}{4}}\right)^{1000-r}+\left(8^{\frac{1}{6}}\right)^{r}$

$={}^{1000}C_r 3^{\frac{1000-r}{2}} 2^{\frac{1}{2}}$

The above term will be rational if exponent of 3 and 2 are integers.

It means $\frac{1000-r}{2}$ and $\frac{r}{2}$ must be integers

The possible set of values of r is {0, 2, 4, ..., 1000}

Hence, number of rational terms is 501.

28. If the fifth term of the expansion $\left(a^{\frac{2}{3}}+a^{-1}\right)^{n}$ does not contain 'a'. Then n is equal to:

(A) 2 (B) 5 (C) 10 (D) None of these.

Ans. :

c. 10

Solution:

$T_5=T_{4+1}$

$={}^{n}C_4\left(a^{\frac{2}{3}}\right)^{n-4}(a^{-1})^4$

$={}^{n}C_4\, a^{\left(\frac{2n-8}{3}-4\right)}$

For this term to be independent of a, we must have

$\frac{2n-8}{3}-4=0$

$\Rightarrow 2n - 20 = 0$

$\Rightarrow n = 10$

29. The coefficient of x^3y^4 in $(2x + 3y^2)^5$ is:

(A) 360 (B) 720 (C) 240 (D) 1080

Ans. :

b. 720

Solution:

Given: $(2x + 3y^2)^5$

Therefore, the general form for the expression $(2x + 3y^2)^5$ is $T_{r+1} = {}^5C_r \times (2x)^r \times (3y^2)^{5-r}$

Hence, $T_{3+1} = {}^5C_3 (2x)^3 \times (3y^2)^{5-3}$

$T_4 = {}^5C_3 (2x)^3 \times (3y^2)^2$

$T4 = {}^5C_3 \times 8x^3 \times 9y^4$

On simplification, we get

$T_4 = 720x^3y^4$

Therefore, the coefficient of x^3y^4 in $(2x + 3y^2)^2$ is 720.

30. The coefficient of x^4 in $\left(\frac{x}{2} - \frac{3}{x^2}\right)$ is:

(A) $\frac{405}{256}$ (B) $\frac{504}{259}$ (C) $\frac{450}{263}$ (D) None of these.

Ans. :

a. $\frac{405}{256}$

Solution:

Suppose x^4 occurs at the $(r + 1)^{th}$ term in the given expansion.

Then, we have

$T_{r+1} = {}^{10}C_r \left(\frac{x}{2}\right)^{10-r} \left(\frac{-3}{2x^2}\right)$

$= (-1)^r\ {}^{10}C_r \frac{3^r}{2^{10-r}} x^{10-r-2r}$

For this term to contain x^4, we must have

$10 - 3r = 4$

$\Rightarrow r = 2$

$\therefore$ Required coefficient $= {}^{10}C_2 \frac{3^2}{2^8} = \frac{10\times9\times9}{2\times2^8} = \frac{405}{256}$

31. The positive integer just greater than $(1 + 0.0001)^{10000}$ is:

(A) 4 (B) 5 (C) 2 (D) 3

Ans. :

d. 3

Solution:

$(1+0.0001)^{10000}$

$=\left(1+\frac{1}{10000}\right)^{10000}$

$=\left(1+\frac{1}{n}\right)^{n}=nc_0(1)^n+nc_1(1)^{n-1}\cdot\frac{1}{n}+nc_2(1)^{n-2}\frac{1}{n^2}+\ \ldots.$

$=1+n\cdot\frac{1}{n}+\frac{n(n-1)}{2}\cdot\frac{1}{n^2}+\ \ldots.$

$=2+\frac{n(n-1)}{2n^2}+\ \ldots.$ > 2 Integer just greater than 2 is 3.

32. Using binomial theorem, the value of $(0.999)^3$ correct to 3 decimal places is:

(A) 0.999 (B) 0.998 (C) 0.997 (D) 0.995

Ans. :

c. 0.997

Solution:

$(0.999)^3 = (1 - 0.001)^3$

$= {}^3C_0 - {}^3C_1\,(0.001) + {}^3C_2\,(0.001)^2 - {}^3C_3\,(0.001)^3$

$= 1 - 0.003 + 3\,(0.000001) - (0.000000001)$

$= 0.997$

33. The number of terms with integral coefficient in the expansion of $\left(17^{\frac{1}{3}}+32^{\frac{1}{2}}\right)^{300}$ is:

(A) 50 (B) 100 (C) 150 (D) 51

Ans. :

d. 51

Solution:

The number of rational terms will be

$1+\frac{300}{\text{L.C.M}(3,2)}$

$=1+\frac{300}{6}$

$=1+50=51$ rational terms.

34. If x4 occurs in the rth term in the expansion of $\left(x^4+\frac{1}{x^3}\right)15$, then what is the value of r?

(A) 4 (B) 8 (C) 9 (D) 10

Ans. :

c. 9

35. The number of non - zero terms in the expansion of $\left(1+3\sqrt{2}\ x\right)^9+\left(1-3\sqrt{2}x\right)^9$ is:

(A) 9 (B) 0 (C) 5 (D) 10

Ans. :

c. 5

Solution:

In the expansion of $(1+3\sqrt{2}\ x)^9 + (1-3\sqrt{2}x)^9$ 2nd, 4th, 6th, 8th and 10th terms get cancelled.

$\therefore$ Number of non - zero terms in $2\left[{}^9C_0 + {}^9C_2(3\sqrt{2}x)^2 + \ \dots\ + {}^9C_8(3\sqrt{2}x)^8\right]$ is 5.

36. The coefficient of x^4 in the expansion of $(1 - 2x)^5$ is equal to:

(A) 40 (B) 320 (C) -320 (D) 80

Ans. :

d. 80

Solution:

General term of $(1 - 2x)^5$ is given by

$T_{r+1} = {}^5C_r\ (-2x)^r$

$= {}^5C_r\ (-2)x^r$

For coefficient of x^4, power of x = 4

$\therefore$ r = 4

$\therefore$ Coefficient pf $x^4 = {}^5C_4\ (-2)^4$

= 5 × 16 = 80

37. The 4th term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^7$ is:

(A) 35x (B) $70x^2$ (C) $35x^2$ (D) 70x

Ans. :

d. 70x

Solution

For the above question

$T_{r+1} = {}^7C_r x^{21-5r} 2^{2r-7}$

For the fourth term, from the end r = 4

$T_{5+1} = {}^7C_4 x^1 2$

= (35)(2)x

= 70x

38. The sum of the coefficients of all the even powers of x in the expansion of $(2x^2 - 3x + 1)^{11}$ is:

(A) 2.6^{10} (B) 3.6^{10} (C) 6^{11} (D) None of the above

Ans. :

b. 3.6^{10}

Solution:

Given equation is $(2x^2 - 3x + 1)^{11}$

$= (2x - 1)^{11}(x - 1)^{11}$

$= (3)^{11}.(2)^{11-1}$

$= 3^{11}.2^{10}$

$= 6^{10}.3$

39. How many terms are there in the expansion of $(1 + 2x + x^2)^{10}$?

(A) 11 (B) 20 (C) 21 (D) 30

Ans. :

c. 21

Solution:

Now, $(1 + 2x + x^2)^{10} = ((1 + x)^2)^{10} = (1 + x)^{20}$

Now, the number of terms in the expansion of $(1 + x)^n$ are n + 1.

Thus, the number of terms in the expansion of $(1 + x)^{20}$ will be 20 + 1 =21.

40. The coefficient of x^{-3} in the expansion of $\left(x - \frac{m}{x}\right)^{11}$ is:

(A) $-924m^7$ (B) $-792m^5$ (C) $-792m^6$ (D) $-330m^7$

Ans. :

d. $-330m^7$

Solution:

Let x^{-3} occur at $(r + 1)^{th}$ term in the given expansion.
Then, we have

$T_{r+1} = {}^{11}C_r\, x^{11-r} \left(\frac{-m}{x}\right)^r$

$= (-1)^r \times {}^{11}C_r\, m^r\, x^{11-r-r}$

For this term to contain x^{-3}, we must have

$= 11 - 2r = -3$

$\Rightarrow r = 7$

Required coefficient $= (-1)^7\, {}^{11}C_7\, m^7$

$= -\frac{11\times10\times9\times8}{4\times3\times2}\, m^7$

$= -330m^7$

41. If the coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^n, n \in N$ are in A.P. then n =

(A) 7 (B) 14 (C) 2 (D) None of these.

Ans. :

a. 7

Solution:

Coefficients of 2nd, 3rd and 4th terms in the expansion of are ${}^nC_1, {}^nC_2, {}^nC_3$.
we have,

$2 \times {}^nC_2 = {}^nC_1 + {}^nC_3$

Dividing both sides by ${}^nC_2$, we get

$2 = \frac{{}^nC_1}{{}^nC_2} + \frac{{}^nC_3}{{}^nC_2}$

$\Rightarrow 2 = \frac{2}{n-1} + \frac{n-2}{3}$

$\Rightarrow 6n - 6 = 6 + n^2 + 2 - 3n$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow n = 7$

42. [AS 1] If $A = \frac{1}{3}B$ and $B = \frac{1}{2}C$, then A : B : C = ..

(A) 1 : 3 : 6 (B) 2 : 3 : 6 (C) 3 : 2 : 6 (D) 3 : 1 : 2

Ans. :

a. 1 : 3 : 6

Solution:

$A = \frac{B}{3} \ldots .(1)$

$B = \frac{C}{2}$

$\Rightarrow C = 2B \ldots ..(2)$

From (1) and (2),

$A : B : C = \frac{B}{3} : B : 2B$

$= \frac{1}{3} : 1 : 2$

$= 1 : 3 : 6$

43. If the sum of the binomial coefficients of the expansion $\left(2x + \frac{1}{x}\right)^n$ is equal to 256, then the term independent of x is:

(A) 1120 (B) 1020 (C) 512 (D) None of these.

Ans. :

a. 1120

Solution:

Suppose (r + 1)th term in the given expansion is independent of x.
Then, we have

$T_{r+1} = {}^nC_r (2x)^{n-r} \left(\frac{1}{x}\right)^r$

$= {}^nC_r (2)^{n-r} x^{n-2r}$

For this term to be independent of x, we must have

$n - 2r = 0$

$\Rightarrow r = \frac{n}{2}$

$\therefore$ Required term $= {}^nC_{\frac{n}{2}}\ 2^{n-\frac{n}{2}} = \frac{n!}{\left[\left(\frac{n}{2}\right)\right]}\ 2^{\frac{n}{2}}$

We know,

Sum of the given expansion = 256

Thus, we have

$2^n.1^n = 256$

$\Rightarrow n = 8$

$\therefore$ Required term $= \frac{8!}{(4)!(4)!}2^4 = 1120$

44. If in the expansion of $\left(x - \frac{1}{3x^3}\right)^9$, the term independent of x is:

(A) T_3 (B) T_4 (C) T_5 (D) None of these.

Ans. :

b. T_4

Solution:

Suppose T_{r+1} is the term in the given expression that is independent of x.

Thus, we have

$T_{r+1} = {}^9C_r\ x^{9-r}\left(\frac{-1}{3x^2}\right)^r$

$= (-1)^r\ {}^9C_r \frac{1}{3^r}\ x^{9-r-2r}$

For this term to be independent of x, we must have

$9 - 3r = 0$

$\Rightarrow r = 3$

Hence, the required term is the 4th term i.e. T_4

45. In the expansion of $\left(\frac{3\sqrt{x}}{3} - \frac{\sqrt{3}}{x}\right)10$, x > 0, the constant term is:

(A) -70 (B) 70 (C) 210 (D) -210

Ans. :

c. 210

46. The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is:

(A) 202 (B) 51 (C) 50 (D) 49

Ans. :

b. 51

Solution:

In the above binomial expansion, the terms at the even places will get eliminated, and we would be left with twice the sum of the terms at odd places.

Hence there will be

$\frac{n}{2} + 1$

$= \frac{100}{2} + 1$

$= 51\text{terms}$

47. The approximate value of $(7.995)^{\frac{1}{3}}$ correct to 4 decimal places is:

(A) 1.9995 (B) 1.9996 (C) 1.9990 (D) 1.9991

Ans. :

a. 1.9995

48. The expansion $\left(\mathrm{x}-\frac{\mathrm{x}^2}{2}\right)^{40}$ is a polynomial of n^{th} degree in x, then n =

(A) 20 (B) 40 (C) 80 (D) 120

Ans. :

c. 80

Solution:

$T_{r+1} = {}^{40}C_r\, x^{40-r}\, x^{2r}\, 2^{-r}$

The power of x = 40 + r

Highest power of x occurs when r = 40(last term)

Hence, highest power of x is 80.

Hence, the polynomial is of degree 80.

49. The 4^{th} term in the expansion of $\left(\sqrt{\mathrm{x}}+\frac{1}{\mathrm{x}}\right)^{12}$ is:

(A) $110\mathrm{x}^{\frac{3}{2}}$ (B) $220\mathrm{x}^{\frac{3}{2}}$ (C) $220\mathrm{x}^2$ (D) $110\mathrm{x}^2$

Ans. :

b. $220\mathrm{x}^{\frac{3}{2}}$

Solution:

Expansion is $\left(\sqrt{\mathrm{x}}+\frac{1}{\mathrm{x}}\right)^{12}$

$\mathrm{T}_{\mathrm{r}+1}=12_{\mathrm{Cr}}\left(\frac{1}{\mathrm{x}}\right)^{\mathrm{r}}\cdot(\sqrt{\mathrm{x}})^{12-\mathrm{r}}=12_{\mathrm{Cr}}\cdot\mathrm{x}^{6-1.5\mathrm{r}}$

4^{th} term is $\mathrm{T}_4=12_{\mathrm{C}3}\cdot\mathrm{x}^{6-1.5\times3}=220.\mathrm{x}^{\frac{3}{2}}$

50. Sum of the coefficients of $(1 - x)^{25}$ is:

(A) -1 (B) 1 (C) 0 (D) 2^{25}

Ans. :

c. 0

Solution:

$(1 - x)^{25} = 1 - {}^{25}C_{1x} + {}^{25}C_2x^2 - {}^{25}C_3x^3 + {}^{25}C_4x^4 - {}^{25}C_5x^5 \ldots - {}^{25}C_{25}x^{25}$

Putting x = 1, we get

$0 = 1 - {}^{25}C_1 + {}^{25}C_2 - {}^{25}C_3 + {}^{25}C_4 - {}^{25}C_5 \ldots - {}^{25}C_{25}$

Hence, sum of coefficients is 0.

51. If $C_0, C_1, C_2, \ldots\ldots, C_n$ are the binomial coefficients, then $2.C_1+2^3.C_3+2^5.C_5+\ldots.$ equals

(A) $\frac{3^n+(-1)^n}{2}$ (B) $\frac{3^n-(-1)^n}{2}$ (C) $\frac{3^n+1}{2}$ (D) $\frac{3^n-1}{2}$

Ans. : b

(b) $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots.. + C_nx^n$

$(1-x)^n = C_0 - C_1x + C_2x^2 - C_3x^3 + \ldots.. + (-1)^n C_nx^n$

$[(1+x)^n - (1-x)^n] = 2\left[C_1x + C_3x^3 + C_5x^5 + \ldots\right]$

$\frac{1}{2}[(1+x)^n - (1-x)^n] = C_1x + C_3x^3 + C_5x^5 + \ldots\ldots$

Put $x = 2$, $2.C_1 + 2^3.C_3 + 2^5.C_5 + \ldots.. = \frac{3^n-(-1)^n}{2}$

52. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots. + C_nx^n$, then $C_0C_2 + C_1C_3 + C_2C_4 + C_{n-2}C_n$ equals

(A) $\frac{(2n)!}{(n+1)!(n+2)!}$ (B) $\frac{(2n)!}{(n-2)!(n+2)!}$ (C) $\frac{(2n)!}{(n)!(n+2)!}$ (D) $\frac{(2n)!}{(n-1)!(n+2)!}$

Ans. : b

(b) We have, $(1+x)^n = C_0 + C_1x + C_2x^2 \ldots. + C_nx^n$

$\left(1+\frac{1}{x}\right)^n = C_0 + C_1.\frac{1}{x} + C_2.\frac{1}{x^2} + \ldots + C_n\left(\frac{1}{x^n}\right)$

on multiplying both expansions, we get

$\frac{(1+x)^{2n}}{x^n} = \sum C_0^2 + x\sum C_0C_1 + x^2\sum C_0C_2 + \ldots.\ \ + x^r\sum C_0C_r + \ldots..$

The various sigma are the coefficient of $x^0, x, x^2, \ldots.., x^r$ in $L.H.S.$ $\frac{(1+x)^{2n}}{x^n}$ or coefficient of $x^n, x^{n+1}, x^{n+2}, \ldots.., x^{n+r}$ in the expansion of $(1+x)^{2n}$ which occur in $T_{n+1}, T_{n+2}, \ldots.$ and are

${}^{2n}C_n, {}^{2n}C_{n+1}, {}^{2n}C_{n+2} \ldots. {}^{2n}C_{n+r}$ etc.

${}^{2n}C_{n+2} = \frac{(2n)!}{(n-2)!(n+2)!}$

53. $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots. =$

(A) $\frac{2^n}{n!}$; for all even values of n

(B) $\frac{2^{n-1}}{n!}$; for all values of n i.e., all even odd values

(C) 0

(D) None of these

Ans. : b

(b) Multiplying each term by $n!$ the question reduces to

$\frac{n!}{1!(n-1)!} + \frac{1}{3!}.\frac{n!}{(n-3)!} + \frac{1}{5!}.\frac{n!}{(n-5)!} + \ldots.$

$= {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots. = 2^{n-1}$.

Thus $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots. = \frac{1}{n!}2^{n-1}$.

54. $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \ldots. + \frac{C_n}{n+1} =$

(A) $\frac{2^n}{n+1}$ (B) $\frac{2^n-1}{n+1}$ (C) $\frac{2^{n+1}-1}{n+1}$ (D) None of these

Ans. : c

(c) Proceeding as above and putting $n+1 = N$.

So given term can be written as

$\frac{1}{N}\{{}^NC_1 + {}^NC_2 + {}^NC_3 + \dots\}$

= $\frac{1}{N}\{2^N - 1\} = \frac{1}{n+1}(2^{n+1} - 1)$

55. $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + \dots$ =

(A) $\frac{2^{n+1}}{n+1}$ (B) $\frac{2^{n+1}-1}{n+1}$ (C) $\frac{2^n}{n+1}$ (D) None of these

Ans. : c

(c) Putting the values of $C_0, C_2, C_4 \dots$, we get $= 1 + \frac{n(n-1)}{3.2!} + \frac{n(n-1)(n-2)(n-3)}{5.4!} + \dots$

$= \frac{1}{n+1}\left[(n+1) + \frac{(n+1)n(n-1)}{3!} + \frac{(n+1)n(n-1)(n-2)(n-3)}{5!} + \dots\right]$

Put $n+1$=N = $\frac{1}{N}\left[N + \frac{N(N-1)(N-2)}{3!} + \frac{N(N-1)\,(N-2)(N-3)(N-4)}{5!} + \dots\right]$

$= \frac{1}{N}\{{}^NC_1 + {}^NC_3 + {}^NC_5 + \dots\}$

$= \frac{1}{N}\{2^{N-1}\} = \frac{2^n}{n+1} \quad \{N = n+1\}$

Trick : Put $n = 1$, then $S_1 = \frac{{}^1C_0}{1} = \frac{1}{1} = 1$

At $n = 2$, $S_2 = \frac{{}^2C_0}{1} + \frac{{}^2C_2}{3} = 1 + \frac{1}{3} = \frac{4}{3}$

Also $(c) \Rightarrow \ S_1 = 1, S_2 = \frac{4}{3}$

56. $C_1 + 2C_2 + 3C_3 + 4C_4 + \dots + nC_n =$

(A) 2^n (B) $n.\ 2^n$ (C) $n.\ 2^{n-1}$ (D) $n.\ 2^{n+1}$

Ans. : c

(c) Trick : Put $n = 1,\ 2,\ 3, \dots$

$S_1 = 1,\ S_2 = 2 + 2 = 4$

Now by alternate (c), put $n = 1,\ 2$

$S_1 = 1.2^0 = 1, S_2 = 2.2^1 = 4$

57. If the sum of the coefficients in the expansion of $(x+y)^n$ is 1024, then the value of the greatest coefficient in the expansion is

(A) 356 (B) 252 (C) 210 (D) 120

Ans. : b

(b) Given $2^n = 1024,\ \ n = 10$

$\therefore$ The greatest coefficient is ${}^{10}C_5 = 252$.

58. The term independent of y in the expansion of $(y^{-1/6} - y^{1/3})^9$ is

(A) 84 (B) 8.4 (C) 0.84 (D) -84

Ans. : d

(d) $(9-r)\left(-\frac{1}{6}\right) + r\left(\frac{1}{3}\right) = 0 \Rightarrow r = 3$

So the term independent of y

= ${}^9C_3(y^{-1/6})^6(-y^{1/3})^3 = -84$.

59. Middle term in the expansion of $(1 + 3x + 3x^2 + x^3)^6$ is

(A) 4^{th} (B) 3^{rd} (C) 10^{th} (D) None of these

Ans. : c

(c) $(1+3x+3x^2+x^3)^6 = \{(1+x)^3\}^6 = (1+x)^{18}$

Hence the middle term is 10^{th}.

60. If the coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270, then $k=$

(A) 1 (B) 2 (C) 3 (D) 4

Ans. : c

(c) $T_{r+1} = {}^5C_r(x^2)^{5-r}\left(\frac{k}{x}\right)^r$

For coefficient of x, $10-2r-r=1 \quad \Rightarrow r=3$

Hence, $T_{3+1} = {}^5C_3(x^2)^{5-3}\left(\frac{k}{x}\right)^3$

According to question, $10\,k^3 = 270 \;\Rightarrow\; k=3$.

61. The value of x in the expression $[x+x^{\log_{10}(x)}]^5$, if the third term in the expansion is $10,00,000$

(A) 10 (B) 11 (C) 12 (D) None of these

Ans. : a

(a) $T_3 = {}^5C_2 . x^2(x^{\log_{10}x})^3 = 10^6$

Put . ${}^5C_2 = 10 \quad [\log_{10}10 = 1]$

If $x=10$, then $10^3.10^{2.1} = 10^5$ is satisfied.

Hence $x=10$.

62. In the expansion of $(5^{1/2}+7^{1/8})^{1024}$, the number of integral terms is

(A) 128 (B) 129 (C) 130 (D) 131

Ans. : b

(b) Here , a power of 2, where as the power of 7 is $\frac{1}{8} = 2^{-3}$

Now first term ${}^{1024}C_0\left(5^{1/2}\right)^{1024} = 5^{512}$ (integer)

And after 8 terms, the 9^{th} term $=$ ${}^{1024}C_8(5^{1/2})^{1016}(7^{1/8})^8$ = an integer

Again, 17^{th} term =${}^{1024}C_{16}(5^{1/2})^{1008}(7^{1/8})^{16}$

= An integer.

Continuing like this, we get an $A.P.$ $1, 9, 17, \ldots, 1025,$

because 1025^{th} term = the last term in the expansion

$= {}^{1024}C_{1024}\left(7^{1/8}\right)^{1024} = 7^{128}$(an integer)

If n is the number of terms of above $A.P.$ we have

$1025 = T_n = 1+(n-1)8 \quad \Rightarrow n=129$.

63. If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a is

(A) $-\frac{7}{9}$ (B) $-\frac{9}{7}$ (C) $\frac{7}{9}$ (D) $\frac{9}{7}$

Ans. : d

(d) $T_{r+1} = {}^{9}C_r(3)^{9-r}(ax)^r = {}^{9}C_r(3)^{9-r}a^r x^r$

$\therefore$ Coefficient of x^r= ${}^{9}C_r 3^{9-r}a^r$

Hence, coefficient of $x^2 = {}^{9}C_2 3^{9-2}a^2$ and coefficient of x^3

= ${}^{9}C_3 3^{9-3}a^3$

So, we must have ${}^{9}C_2 3^7 a^2 = {}^{9}C_3 3^6 a^3$

==> $\frac{9.8}{1.2}.3 = \frac{9.8.7}{1.2.3}.a$

$\Rightarrow\ a = \frac{9}{7}$.

64. If the coefficients of second, third and fourth term in the expansion of $(1+x)^{2n}$ are in $A.P.$, then $2n^2 - 9n + 7$ is equal to

(A) -1 (B) 0 (C) 1 (D) $\frac{3}{2}$

Ans. : b

(b) $T_2 = {}^{2n}C_1\ x$, $T_3 = {}^{2n}C_2\ x^2$, $T_4 = {}^{2n}C_3\ x^3$

Coefficient of T_2,T_3,T_4 are in $A.P.$

==> $2.{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

==> $2\frac{2n!}{2\,!\,(2n-2)\,!} = \frac{2n!}{(2n-1)\,!} + \frac{2n!}{3\,!\,(2n-3)\,!}$

==> $\frac{2\ .\ 2n(2n-1)}{2} = 2n + \frac{2n(2n-1)(2n-2)}{6}$

==> $n(2n-1) = n + \frac{(n)(2n-1)(2n-2)}{6}$

==> $6(2n^2 - n) = 6n + 4n^3 - 6n^2 + 2n$

==> $6n(2n-1) = 2n(2n^2 - 3n + 4)$

==> $6n - 3 = 2n^2 - 3n + 4$

==> $0 = 2n^2 - 9n + 7$

==> $2n^2 - 9n + 7 = 0$.

65. The coefficient of x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + \dots\dots\dots + (1+x)^{30}$ is

(A) ${}^{51}C_5$ (B) ${}^{9}C_5$ (C) ${}^{31}C_6 - {}^{21}C_6$ (D) ${}^{30}C_5 + {}^{20}C_5$

Ans. : c

(c) $(1+x)^{21} + (1+x)^{22} + \dots. + (1+x)^{30}$

$= (1+x)^{21}\left[\frac{(1+x)^{10}-1}{(1+x)-1}\right]$

= $\frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$

$\therefore$ Coefficient of x^5 in the given expression

= Coefficient of x^5 in $\left\{\frac{1}{x}[(1+x)^{31} - (1+x)^{21}]\right\}$

= Coefficient of x^6 in $[(1+x)^{31} - (1+x)^{21}]$ = ${}^{31}C_6 - {}^{21}C_6$.

66. If coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the binomial expansion of $(1+x)^n$ are in $A.P.$, then n^2-9n is equal to

(A) -7 (B) 7 (C) 14 (D) -14

Ans. : d

(d) Coefficients of 2^{nd}, 3^{rd} and 4^{th} terms are respectively ${}^nC_1$, ${}^nC_2$ and ${}^nC_3$ are in $A.P.$

==> $2.{}^nC_2 = {}^nC_1 + {}^nC_3$

==> $\frac{2n!}{2\,!(n-2)\,!}$

= $\frac{n!}{(n-1)!} + \frac{n!}{3!\,(n-3)\,!}$ On solving, $n^2-9n+14=0$

$\Rightarrow\ n^2-9n=-14$.

67. If x^moccurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{2n}$, then the coefficient of x^m is

(A) $\frac{(2n)!}{(m)!\,(2n-m)!}$ (B) $\frac{(2n)!\,3!\,3!}{(2n-m)!}$ (C) $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ (D) None of these

Ans. : c

(c) $T_{r+1} = {}^{2n}C_r x^{2n-r}\left(\frac{1}{x^2}\right)^r$

= ${}^{2n}C_r x^{2n-3r}$,

This contains x^m, if $2n-3r=m\ i.e.$ if $r=\frac{2n-m}{3}$

Coefficient of $x^m = {}^{2n}C_r,\ r=\frac{2n-m}{3}$ = $\frac{2n!}{(2n-r)!r!}$

=$\frac{2n!}{\left(2n-\frac{2n-m}{3}\right)!\left(\frac{2n-m}{3}\right)!}$

= $\frac{2n!}{\left(\frac{4n+m}{3}\right)!\left(\frac{2n-m}{3}\right)!}$.

68. If the coefficients of 5^{th}, $6^{th}$and 7^{th} terms in the expansion of $(1+x)^n$be in $A.P.$, then $n=$

(A) 7 only (B) 14 only (C) 7 or 14 (D) None of these

Ans. : c

(c) Coefficient of $T_5 = {}^nC_4, T_6 = {}^nC_5$ and $T_7 = {}^nC_6$

According to the condition, $2\ {}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow\ 2\left[\frac{n!}{(n-5)!5!}\right] = \left[\frac{n!}{(n-4)\,!\,4\,!} + \frac{n!}{(n-6)\,!\,6\,!}\right]$

$\Rightarrow\ 2\left[\frac{1}{(n-5)\,5}\right] = \left[\frac{1}{(n-4)(n-5)} + \frac{1}{6\times5}\right]$

After solving, we get $n=7$ or 14.

69. In the expansion of $\left(\frac{a}{x}+bx\right)^{12}$,the coefficient of x^{-10} will be

(A) $12a^{11}$ (B) $12b^{11}a$ (C) $12a^{11}b$ (D) $12a^{11}b^{11}$

Ans. : c

(c) $\left[\exp\frac{a}{x}\right]^{12-r} + [\exp bx]^r = -10$

$\Rightarrow -12+r+r=-10$

$\Rightarrow r=1$

Then coefficient of x^{-10} is ${}^{12}C_1(a)^{11}(b)^1=12a^{11}b$.

70. The first 3 terms in the expansion of $(1+ax)^n$ $(n\neq 0)$ are $1, 6x$ and $16x^2$. Then the value of a and n are respectively

(A) 2 and 9 (B) 3 and 2 (C) $2/3$ and 9 (D) $3/2$ and 6

Ans. : c

(c) $T_1={}^nC_0=1$ .....(i)

$T_2={}^nC_1ax=6x$ .....(ii)

$T_3={}^nC_2(ax)^2=16x^2$ .....(iii)

From (ii), $\frac{n!}{(n-1)!}a=6$

==> $n\,a\,=\,6$ .....(iv)

From (iii), $\frac{n(n-1)}{2}\,a^2=16$ .....(v)

Only (c) is satisfying equation (iv) and (v).

71. If the third term in the binomial expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m is

(A) 2 (B) $1/2$ (C) 3 (D) 4

Ans. : b

(b) We have $(1+x)^m=1+mx+\frac{m(m-1)}{2!}x^2+\ldots$

By hypothesis, $\frac{m(m-1)}{2}x^2=-\frac{1}{8}x^2$

==> $4m^2-4m=-1$

==> $(2m-1)^2=0$

$\Rightarrow m=\frac{1}{2}$.

72. If x^4 occurs in the r^{th} term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then $r=$

(A) 7 (B) 8 (C) 9 (D) 10

Ans. : c

(c) $T_r=\,{}^{15}C_{r-1}(x^4)^{16-r}\left(\frac{1}{x^3}\right)^{r-1}=\,{}^{15}C_{r-1}x^{67-7r}$

==> $67-7r=4$

$\Rightarrow r=9$.

73. If coefficient of $(2r+3)^{th}$ and $(r-1)^{th}$ terms in the expansion of $(1+x)^{15}$ are equal, then value of r is

(A) 5 (B) 6 (C) 4 (D) 3

Ans. : a

(a) ${}^{15}C_{2r+2}={}^{15}C_{r-2}$

But ${}^{15}C_{2r+2} = {}^{15}C_{15-(2r+2)} = {}^{15}C_{13-2r}$
==> ${}^{15}C_{13-2r} = {}^{15}C_{r-2}$
$\Rightarrow r = 5.$

74. In $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ if the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is $\frac{1}{6}$, then $n=$

(A) 7 (B) 8 (C) 9 (D) None of these

Ans. : c

(c) $\frac{1}{6} = \frac{{}^nC_6\left(2^{1/3}\right)^{n-6}\left(3^{-1/3}\right)^6}{{}^nC_{n-6}\left(2^{1/3}\right)^6\left(3^{-1/3}\right)^{n-6}}$ or $6^{-1} = 6^{-4}.6^{n/3} = 6^{n/3-4}$
$\frac{n}{3} - 4 = -1$
$\Rightarrow n = 9.$

75. If the coefficients of r^{th} term and $(r+4)^{th}$ term are equal in the expansion of $(1+x)^{20}$, then the value of r will be

(A) 7 (B) 8 (C) 9 (D) 10

Ans. : c
(c) ${}^{20}C_{r-1} = {}^{20}C_{r+3}$
$\Rightarrow 20 - r + 1 = r + 3$
$\Rightarrow r = 9.$

76. If the ratio of the coefficient of third and fourth term in the expansion of $\left(x - \frac{1}{2x}\right)^n$ is $1:2$, then the value of n will be

(A) 18 (B) 16 (C) 12 (D) -10

Ans. : d

(d) $T_3 = {}^nC_2(x)^{n-2}\left(-\frac{1}{2x}\right)^2$ and $T_4 = {}^nC_3(x)^{n-3}\left(-\frac{1}{2x}\right)^3$
But according to the condition,
$\frac{-n(n-1)\times 3\times 2\times 1\times 8}{n(n-1)(n-2)\times 2\times 1\times 4} = \frac{1}{2}$
$\Rightarrow n = -10$

77. 6^{th} term in expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is

(A) $\frac{4580}{17}$ (B) $-\frac{896}{27}$ (C) $\frac{5580}{17}$ (D) None of these

Ans. : b
(b) Applying $T_{r+1} = {}^nC_r x^{n-r} a^r$ for $(x+a)^n$

Hence $T_6 = {}^{10}C_5(2x^2)^5\left(-\frac{1}{3x^2}\right)^5$

$= -\frac{10!}{5!5!}32 \times \frac{1}{243} = -\frac{896}{27}$

78. The last digit in 7^{300} is

(A) 7 (B) 9 (C) 1 (D) 3

Ans. : c

(c) We have $7^2 = 49 = 50 - 1$

Now, $7^{300} = (7^2)^{150} = (50-1)^{150}$

= ${}^{150}C_0(50)^{150}(-1)^0 + {}^{150}C_1(50)^{149}(-1)^1 + \ldots\ldots + {}^{150}C_{150}(50)^0(-1)^{150}$

Thus the last digits of 7^{300} are ${}^{150}C_{150}.1.1\ i.e., 1$.

79. The number of non-zero terms in the expansion of $(1+3\sqrt{2}x)^9 + (1-3\sqrt{2}x)^9$ is

(A) 9 (B) 0 (C) 5 (D) 10

Ans. : c

(c) Given expression

$= 2\left[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4 + {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8\right]$

The number of non-zero terms is 5.

*** Given section consists of questions of 2 marks each.** **[8]**

80. Using binomial theorem, evaluate: $(101)^4$

Ans. : $(101)^4 = (100 + 1)^4$

Using binomial theorem, we have

$(100+1)^4 = {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4$

= $(100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + 1$

= 100000000 + 4000000 + 60000 + 400 + 1

= 104060401

81. Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Ans. : $(1.1)^{10000} = (1 + 0.1)^{10000}$.

= $1 + {}^{10000}C_1(0.1) + {}^{10000}C_2(0.1)^2 + {}^{10000}C_3(0.1)^3 + \ldots$

= 1 + 10000 (0.1) + other positive numbers

= 1 + 1000 + other positive numbers

Which is greater than 1000.

Thus (1.1)10000 > 1000

82. Which term in the expansion of $\left\{\left(\frac{x}{\sqrt{y}}\right)^{\frac{1}{3}} + \left(\frac{y}{x^{\frac{1}{3}}}\right)^{\frac{1}{2}}\right\}^{21}$ contains x and y to one and the same power?

Ans. :

$\mathrm{T_n = T_{r+1} = {}^nC_r x^{n-r} y^r}$

$= {}^{21}\mathrm{C_r}\left(\left(\frac{x}{\sqrt{y}}\right)^{\frac{1}{3}}\right)^{21-r}\left(\left(\frac{y}{x^{\frac{1}{3}}}\right)^{\frac{1}{2}}\right)^{r}$

$= {}^{21}C_r \left(\frac{x^{7-\frac{r}{3}}}{y^{\frac{7}{2}-\frac{r}{6}}}\right)\frac{y^{\frac{r}{2}}}{x^{\frac{r}{6}}}$

$\frac{x^{7-\frac{r}{3}-\frac{r}{6}}}{y^{\frac{7}{2}-\frac{r}{6}-\frac{r}{2}}}$

$\Rightarrow x^{\frac{42-2r-r}{6}} = y^{\frac{21-r-3r}{6}}$

Since x and y have same power

$\frac{42-3r}{6} = \frac{-(21-4r)}{6}$

$42 + 21 = 4r + 3r$

$63 = 7r$

$r = 9$

Term is 10th

$(t_n = t_{r+1})$

83. Find the coefficient of:

x in the expansion of $(1-2x^3+3x^5)\left(1+\frac{1}{x}\right)^8$.

Ans. :

$(1-2x^2+3x^3)\left(1+\frac{1}{x}\right)^2$

$= (1-2x^3+3x^5)\left({}^8C_0 + {}^8C_1\left(\frac{1}{x}\right) + {}^8C_2\left(\frac{1}{x}\right)^2 + {}^8C_3\left(\frac{1}{x}\right)^3\right.$

$\left. + {}^8C_4\left(\frac{1}{x}\right)^4 + {}^8C_5\left(\frac{1}{x}\right)^5 + {}^8C_6\left(\frac{1}{x}\right)^6\right)$

x occurs in the above expression at $-2x^3.{}^8C_2\left(\frac{1}{x^2}\right) + 3x^5.{}^8C_4\left(\frac{1}{x}\right)4.$

$\therefore$ Coefficient of $x = -2\left(\frac{8!}{2!6!}\right) + 3\left(\frac{8!}{4!4!}\right) = -56 + 210 = 154$

*** Given section consists of questions of 3 marks each.** **[48]**

84. Expand the given expression $(1 - 2x)^5$

Ans. : Using binomial theorem for the expansion of $(1 - 2x)^5$ we have

$(1-2x)^5 = {}^5C_0 + {}^5C_1(-2x) + {}^5C_2(-2x)^2 + {}^5C_3(-2x)^3 + {}^5C_4(-2x)^4 + {}^5C_5(-2x)^5$

$= 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

85. Expand the given expression $\left(x+\frac{1}{x}\right)^6$

Ans. : Using binomial theorem for the expansion of $\left(x+\frac{1}{x}\right)^6$ we have

$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3$

$+ {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{6}\right)^6$

$= x^6 + 6 \cdot x^5 \cdot \frac{1}{x} + 15 \cdot 4x^4 \cdot \frac{1}{x^2} + \ 20 \cdot x^3 \cdot \frac{1}{x^3} + 15 \cdot x^2 \cdot \frac{1}{x^4} + 6 \cdot x \cdot \frac{1}{x^5} + \frac{1}{x^6}$

$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

86. Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$

Ans. : $(a + b)^4 = [{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4]$

$and\ (a-b)^4 = [{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4]$

$\therefore\ (a+b)^4 - (a-b)^4 = 2\left[{}^4C_1a^3b + {}^4C_3ab^3\right]$

$= 2\left[4a^3b + 4ab^3\right] = 8ab\left[a^2+b^2\right]$

$\therefore \left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8.\sqrt{3}.\sqrt{2}\left[\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right]$

$= 8.\sqrt{3}.\sqrt{2}\,[3+2] = 40.\sqrt{3}.\sqrt{2} = 40\sqrt{6}$

87. Show that $9^{n+1} - 8n - 9$ is divisible by 64 whenever n is a positive integer.

Ans. : We have $9^{n+1} = (1 + 8)^{n+1}$

$= {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$= 1 + (n+1)\times 8 + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$= 1 + 8n + 8 + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots\ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$= 9 + 8n + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots\ldots\ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$\Rightarrow 9^{n+1} - 8n - 9 = {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$= 64[{}^{n+1}C_2 + {}^{n+1}C_3 \cdot 8 + \ldots + {}^{n+1}C_{n+1} \cdot 8^{n+1}]$

which show that 9^{n+1} - 8^{n-9} is divisible by 64 wherever n is a positive integer

88. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$.

Ans. :

89. If 3rd, 4th, 5th and 6th terms in the expansion of be $(x+a)^n$ respectively a, b, c and d. prove that $\frac{b^2-ac}{c^2-bd} = \frac{5a}{3c}$.

Ans. :

$(x+a)^n$

Then,

$T_3 = a = {}^nC_2x^{n-2}\alpha^2 = \frac{n(n-1)}{2}x^{n-2}\alpha^2$

$T_4 = b = {}^nC_3x^{n-3}\alpha^3 = \frac{n(n-1)(n-2)}{6}x^{n-3}\alpha^3$

$T_5 = c = {}^nC_4x^{n-4}\alpha^4 = \frac{n(n-1)(n-2)(n-3)}{24}x^{n-4}\alpha^4$

$T_6 = d = {}^nC_5x^{n-5}\alpha^5 = \frac{n(n-1)(n-2)(n-3)(n-4)}{120}x^{n-5}\alpha^5$

Now,

$$\frac{T_4}{T_3}=\frac{b}{a}=\frac{\frac{n(n-1)(n-2)x^{n-3}\alpha^3}{6}}{\frac{n(n-1)}{2}x^{n-2}\alpha^2}=\left(\frac{n-2}{3}\right)\cdot\frac{\alpha}{x}\ldots(i)$$

Similarly,

$$\frac{T_5}{T_4}=\frac{c}{b}=\frac{\frac{n(n-1)(n-2)(n-3)x^{n-4}\alpha^4}{24}}{\frac{n(n-1)(n-2)(n-3)}{6}x^{n-3}\alpha^3}=\left(\frac{n-3}{4}\right)\cdot\frac{\alpha}{x}\ldots(ii)$$

and,

$$\frac{T_6}{T_5}=\frac{d}{c}=\frac{\frac{n(n-1)(n-2)(n-3)(n-4)x^{n-5}\alpha^5}{120}}{\frac{n(n-1)(n-2)(n-3)}{24}x^{n-4}\alpha^4}=\left(\frac{n-4}{5}\right)\cdot\frac{\alpha}{x}\ldots(iii)$$

Again, dividing (i) by (ii) and (ii) by (iii), we get

$$\frac{\frac{b}{a}}{\frac{c}{b}}=\frac{\left(\frac{n-2}{3}\right)\cdot\frac{\alpha}{x}}{\left(\frac{n-3}{4}\right)\cdot\frac{\alpha}{x}}=\frac{4(n-2)}{3(n-3)}$$

$$\Rightarrow\frac{b^2}{\alpha c}=\frac{4(n-2)}{3(n-3)}\ldots(iv)$$

and,

$$\frac{\frac{c}{b}}{\frac{d}{c}}=\frac{\left(\frac{n-3}{4}\right)\cdot\frac{\alpha}{x}}{\left(\frac{n-4}{5}\right)\cdot\frac{\alpha}{x}}=\frac{5(n-3)}{4(n-4)}$$

$$\Rightarrow\frac{c^2}{bd}=\frac{5(n-3)}{4(n-4)}\ldots(v)$$

Now subtracting 1 from both sides of quation (iv) and (v) as:

$$\Rightarrow\frac{b^2}{\alpha c}-1=\frac{4(n-2)}{3(n-3)}-1$$

$$\Rightarrow\frac{b^2-ac}{\alpha c}=\frac{n+1}{3(n-3)}\ldots(vi)$$

$$\Rightarrow\frac{c^2}{bd}-1=\frac{5(n-3)}{4(n-4)}-1$$

$$\Rightarrow\frac{c^2-bd}{bd}=\frac{n+1}{4(n-4)}\ldots(vii)$$

Again, on diving (vi) by (vii), we get

$$\frac{\frac{b^2-ac}{ac}}{\frac{c^2-bd}{bd}}=\frac{\frac{n+1}{3(n-3)}}{\frac{n+1}{4(n)-4}}$$

$$\Rightarrow\frac{b^2-ac}{c^2-bd}\times\frac{bd}{ac}=\frac{4(n-4)}{3(n-3)}\ldots(viii)$$

On multipiying (v) by (viii), we get

$$\Rightarrow\frac{b^2-ac}{c^2-bd}\times\frac{bd}{ac}\times\frac{c^2}{bd}=\frac{4(n-4)}{3(n-3)}\times\frac{5(n-3)}{4(n-4)}$$

$$\Rightarrow\frac{b^2-ac}{c^2-bd}\times\frac{c}{a}=\frac{5}{3}$$

$$\Rightarrow\frac{b^2-ac}{c^2-bd}=\frac{5a}{3c}\ \ldots(IX)$$

By Equation (IX), it is proved that,

L.H.S = R.H.S

90. Using binomial theorem write down the expansions of the following:

$\left(ax - \frac{b}{x}\right)^6$

Ans. :

The expansion of $(x + y)^n$ has n + 1 terms so the expansion of $\left(ax - \frac{b}{x}\right)^6$ has 7 term Using binomial theorem to expand, we get

$$\left(ax - \frac{b}{x}\right)^6 = {}^6C_0(ax)^6\left(\frac{b}{x}\right) - {}^6C_1(ax)^5\left(\frac{b}{x}\right) + {}^6C_2(ax)^4\left(\frac{b}{x}\right)^2$$
$$- {}^6C_3(ax)^3\left(\frac{b}{x}\right)^3 + {}^6C_4(ax)^2\left(\frac{b}{x}\right)^4 - {}^6C_5(ax)\left(\frac{b}{x}\right)^5$$
$$+ {}^6C_6(ax)^0\left(\frac{b}{x}\right)^6$$

$$= a^6x^6 - 6a^5x^5\frac{b}{x} + 15a^4x^4\frac{b^2}{x^2}$$
$$- 20a^3b^3 + 15a^2\frac{b^4}{x^2} - 6a\frac{b^5}{x}^4 + \frac{b^6}{x^6}$$

$$= a^6x^6 - 6a^5x^4b\frac{b}{x} + 15a^4b^2x^2 - 20a^3b^3 + 15\frac{a^2b^4}{x^2} - 6\frac{ab^5}{x^4} + \frac{b^6}{x^6}$$

91. Find the 4th term from the end in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Ans. :

$T_N = T_{r+1} = (-1)^r\ {}^nC_2x^{n-r}y^r$

4th term from the end = 7th term from beginning

$N = 7, r + 6, n = 9, x = \frac{4x}{5}, y = \frac{5}{2x}$

$T_7 = T_{6+1} = (-1)^6\ {}^9C_6\left(\frac{4x}{5}\right)^3\left(\frac{5}{2x}\right)^6 = \frac{9\times8\times7}{3\times2} \times \frac{4^3\times5^6}{5^3\times2^6} \times \frac{x^3}{x^6}$

$\frac{9\times8\times7\times5^3}{6\times x^3} = \frac{9\times8\times7\times125}{6\times x^3} = \frac{10500}{x^3}$

92. Find the middle term in the expansion of:

$\left(\frac{2}{3}x - \frac{3}{2x}\right)^{20}$

Ans. :

Here, n = 20 which is an even number so, $\left(\frac{20}{2} + 1\right)^{th}$ i.e., 11th term is the middle term.

We know that,

$T_n = T_{r+1} = (-1)^r\ {}^nC_rx^{n-r}y^r$

$n = 20, r = 10, x = \frac{2}{3}x, y = \frac{2}{3x}$

$T_{11} = T_{10+1} = (-1)^{10}\ {}^{20}C_{10}\left(\frac{2}{3}x\right)^{10}\left(\frac{3}{2x}\right)^{10}$

$= {}^{20}C_{10}\frac{2^{10}}{3^{10}} \times \frac{3^{10}}{2^{10}} \times \frac{x^{10}}{x^{10}}$

$= {}^{20}C_{10}$

93. Find the middle terms(s) in the expansion of:

$(1+3x+3x^2+x^3)^{2n}$

Ans. :

$(1+3x+3x^2+x^3)^{2n}$

$=(1+x)^{6n}$

Here, n is an even number.

Middle term $= \left(\frac{6n}{2}+1\right) = (3n+1)$

Now, we have,

T_{3n+1}

$= {}^{6n}C_{3n}x^{3n}$

$= \frac{6n!}{3n!}x^{3n}$

94. If in the expansion of $(1+x)^n$ the coefficients of three consecutive terms are 56, 70 and 56, then find n and the position of the terms of these coefficients.

Ans. :

Suppose $r^{th}, (r+1)^{th}$ and $(r+2)^{th}$ terms are the three consecutive terms.

Their respective coefficients are ${}^nC_{r-1}, {}^nC_r$ and ${}^nC_{r+1}$.

We have,

${}^nC_{r-1} = {}^nC_{r+1} = 56$

$\Rightarrow r-1+r+1=n$

$\Rightarrow 2r=n$

$\Rightarrow r=\frac{n}{2}$

Now,

${}^nC_{\frac{n}{2}} = 70$ and ${}^nC_{\left(\frac{n}{2}-1\right)} = 56$

$\Rightarrow \frac{{}^nC_{\frac{n}{2}-1}}{{}^nC_{\frac{n}{2}}} = \frac{56}{70}$

$\Rightarrow \frac{\frac{n}{2}}{\left(\frac{n}{2}+1\right)} = \frac{8}{10}$

$\Rightarrow 5n=4n+8$

$\Rightarrow n=8$

So, $r=\frac{n}{2}=4$

Thus, the required term 4th, 5th and 6th.

95. Find the value of r, if the coefficients of $(2r + 4)^{th}$ and $(r - 2)^{th}$ terms in the expansion of $(1 + x)^{18}$ are equal.

Ans. : Given expression is $(1 + x)^{18}$

Now, $(2r + 4)^{th}$ term, i.e., $T_{(2r+3)+1}$

$\therefore\ T_{(2r+3)+1} = {}^{18}C_{2r+3}(x)^{2r+3}$

And $(r - 2)^{th}$ term, i.e., $T_{(r-3)+1}$

$\therefore\ T_{(r-3)+1} = {}^{18}C_{r-3}x^{r-3}$

According to the question,

${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow 2r + 3 + r - 3 = 18\ \ [\because\ {}^{n}C_{x} = {}^{n}C_{y} \Rightarrow x + y = n]$

$\Rightarrow 3r = 18\ \therefore r = 6$

96. If p is a real number and if the middle term in the expansion of $\left(\frac{p}{2} + 2\right)^8$ is 1120, find P.

Ans. : Give expression is $\left(\frac{P}{2} + 2\right)^8$

Number of term = 8 + 1 = 9 (odd)

$\therefore$ Middle term $= \frac{9+1}{2}$th term = 5^{th} term

$\therefore T_5 = T_{4+1} = {}^{8}C_4\left(\frac{P}{2}\right)^{8-4}(2)^4$

$= {}^{8}C_4\frac{P^4}{2^4} \times 2^4 = {}^{8}C_4P^4$

Now ${}^{8}C_4P^4 = 1120 \Rightarrow \frac{8\times7\times6\times5}{4\times3\times2\times1}P^4 = 1120$

$\Rightarrow 70P^4 = 1120$

$\Rightarrow P^4 = \frac{1120}{70} = 16 \Rightarrow P^4 = 2^2 \Rightarrow P = \pm2$

Hence, the required value of $P = \pm2$.

97. If the coefficient of second, third and fourth terms in the expansion of $(1 + x)^{2n}$ are in A.P. Show that $2n^2 - 9n + 7 = 0$.

Ans. : Given expression is $(1 + x)^{2n}$

Now, coefficient of $2^{nd}, 3^{rd}$ and 4^{th} terms are ${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$, respectively.

Given that, ${}^{2n}C_2$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are in A.P.

Then, $2.{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow 2\left[\frac{2n(2n-1)(2n-2)!}{2\times1\times(2n-2)!}\right] = 2n + \frac{2n(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!}$

$\Rightarrow n(2n-1) = n + \frac{n(2n-1)(n-1)}{3}$

$\Rightarrow 3(2n-1) = 3 + (2n^2 - 3n + 1)$

$\Rightarrow 6n - 3 = 2n^2 - 3n + 4 \Rightarrow 2n^2 - 9n + 7 = 0$

98. Find the coefficient of x^{15} in the expansion of $(x - x^2)^{10}$.

Ans. : The given expression is $(x-x^2)^{10}$

General Term $T_{r+1} = {}^nC_r x^{n-r} y^r$

$= {}^{10}C_r (x)^{10-r}(-x^2)^r = {}^{10}C_r (x)^{10-r}(-1)^r.(x^2)^r$

$= (-1)^r.{}^{10}C_r (x)^{10-r+2r} = (-1)^r.{}^{10}C_r (x)^{10+r}$

To find the coefficient of x^{15}, Put $10+r=15 \Rightarrow r=5$

$\therefore$ Hence, the required coefficient = -252

99. Find n in the binomial $\left(3\sqrt{2}+\frac{1}{3\sqrt{3}}\right)^n$ if the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is $\frac{1}{6}$.

Ans. : Give expression is $\left(3\sqrt{2}+\frac{1}{3\sqrt{3}}\right)^n$

Now, 7th term from beginning, $T_7 = T_{6+1} = {}^nC_6 (3\sqrt{2})^{n-6}\left(\frac{1}{3\sqrt{3}}\right)^6 \ldots(i)$

And 7th term from end is same as 7th term from the beginning of $\left(\frac{1}{3\sqrt{3}}+3\sqrt{2}\right)^n$

i.e., $T_7 = {}^nC_6 \left(\frac{1}{3\sqrt{3}}\right)^{n-6}(3\sqrt{2})^6 \quad \ldots.(ii)$

Given that, $\frac{{}^nC_6(3\sqrt{2})^{n-6}\left(\frac{1}{3\sqrt{3}}\right)^6}{{}^nC_6\left(\frac{1}{3\sqrt{3}}\right)^{n-6}(3\sqrt{2})^6} = \frac{1}{6}$

$\Rightarrow \frac{(3\sqrt{2})^{n-12}}{\left(\frac{1}{3\sqrt{3}}\right)^{n-12}} = \frac{1}{6} \Rightarrow (3\sqrt{2}\ \ 3\sqrt{3})^{n-12} = 6^{-1} \Rightarrow 6^{\frac{n-12}{3}} = 6^{-1}$

$\Rightarrow \frac{n-12}{3} = -1 \Rightarrow n = 9$

*** Given section consists of questions of 5 marks each.** **[65]**

100. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively.

Ans. :

101. Find the coefficient of x^5 in the product $(1 + 2x)^6 (1 - x)^7$ using binomial theorem.

Ans. :

102. Evaluate the following:

$(0.99)^5 + (1.01)^5$

Ans. :

$(0.99)^5 + (1.01)^5$

$= (1-.01)^5 + (1+.01)^5$

$= 2\left[{}^5C_1 + {}^5C_3(.01)^2 + {}^5C_5(.01)^5\right]$

$= 2\left[5 + 10 \times \frac{1}{10^4} + \frac{1}{10^{10}}\right]$

$= 2\left[5 + \frac{1}{1000} + \frac{1}{10^{10}}\right]$

$= 2.0020001$

103. Find a, b and n in the expansion of $(a+b)^n$, if the first three terms in the expansion are 729, 7290 and 30375 respectively.

Ans. :

We have,

T_1 = 729, T_2 = 7290 and T_3 = 30375

According to the quation,

${}^nC_0 a^n b^0 = 729$

$\Rightarrow a^n = 729$

$\Rightarrow a^n = 3^6$

${}^nC_1 a^{n-1} b^1 = 7290$

${}^nC_2 x^{n-2} a^2 = 30375$

Also,

$\Rightarrow \frac{{}^nC_2 x^{n-2} a^2}{{}^nC_1 x^{n-1} a^1} = \frac{30375}{7290}$

$\Rightarrow \frac{n-1}{2} \times \frac{b}{a} = \frac{25}{6} \ldots (i)$

$\Rightarrow \frac{\left(n-1\right)b}{a} = \frac{25}{3}$

And,

$\frac{{}^nC_1 a^{n-1} b^1}{{}^nC_0 a^n b^0} = \frac{7290}{729}$

$\Rightarrow \frac{nb}{a} = \frac{10}{1} \ldots (ii)$

On dividing (ii) by (i), we get

$\frac{\frac{nd}{a}}{\frac{(n-1)b}{a}} = \frac{10 \times 3}{25}$

$\Rightarrow \frac{n}{n-1} = \frac{6}{5}$

$\Rightarrow n = 6$

Since, $a^6 = 3^6$

Hence, $a = 3$

Now,

$\frac{nb}{a} = 10$

$\Rightarrow b = 5$

104. If the term from x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, find the value of k.

Ans. :

Let (r + 1)th term, in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$, be free from x and be equal to T_{r+1}.

Then,

$T_{r+1}={}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{-k}{x^2}\right)^r={}^{10}C_r x^{5-\frac{5r}{2}}(-k)^r \ldots(i)$

If T_{r+1} is independent of x, then

$5-\frac{5r}{2}=0$

$\Rightarrow r=2$

Putting r = 2 in (i), we obtain

$T_3={}^{10}C_2(-k)^2=45k^2$

But it is given that the value of the term free x is 405.

$\therefore 45k^2=405$

$\Rightarrow k^2=9$

$\Rightarrow k^2=\pm 3$

Hence, the value of k is ± 3.

105. Find n in the binomial $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, if the ratio of 7th term from the beginning to the 7th term form the end is $\frac{1}{6}$.

Ans. :

In the binomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, $\left[(n+1)-7+1\right]^{th}$ i.e., $(n-5)^{th}$ term from the beginning is the 7th term from the end.

Now,

$T_7={}^nC_6\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6={}^nC_6\times 2^{\frac{n}{3}-2}\times\frac{1}{3^2}$

And,

$T^{n-5}={}^nC_{n-6}\left(\sqrt[3]{2}\right)^6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}={}^nC_6\times 2^2\times\frac{1}{3^{\frac{n}{3}-2}}$

It is given that,

$\frac{T^7}{T^{n-5}}=\frac{1}{6}$

$\Rightarrow \frac{{}^nC_6\times 2^{\frac{n}{3}-2}\times\frac{1}{3^2}}{{}^nC_6\times 2^2\times\frac{1}{3^{\frac{n}{3}-2}}}=\frac{1}{6}$

$\Rightarrow 2^{\frac{n}{3}-2-2}\times 3^{\frac{n}{3}-2-2}=\frac{1}{6}$

$\Rightarrow \left(\frac{1}{6}\right)^{4-\frac{n}{3}}=\frac{1}{6}$

$\Rightarrow 4-\frac{n}{3}=1$

$\Rightarrow n = 9$

Hence, the vaue of n is 9.

106. If the seventh term from the beginning and in the binomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ are equal, is the 7th term from the end.

Ans. :

In the binomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n, \left[(n+1)-7+1\right]^{th}$ i.e., $(n-5)^{th}$ term from the beginning is the 7th term from the end.

Now,

$$T_7 = {}^nC_6\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6 = {}^nC_6 \times 2^{\frac{n}{3}-2} \times \frac{1}{3^2}$$

And,

$$T_{n-5} = {}^nC_{n-6}\left(\sqrt[3]{2}\right)^6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} = {}^nC_6 \times 2^2 \times \frac{1}{3^{\frac{n}{3}-2}}$$

It is given that,

$$T_7 = T_{n-5}$$

$$\Rightarrow {}^nC_6 \times 2^{\frac{n}{3}-2} \times \frac{1}{3^2}$$

$$\Rightarrow {}^nC_6 \times 2^2 \times \frac{1}{3^{\frac{n}{3}-2}}$$

$$\Rightarrow \frac{2^{\frac{n}{3}-2}}{2^2} = \frac{3^2}{3^{\frac{n}{3}-2}}$$

$$\Rightarrow (6)^{\frac{n}{3}-2} = 6^2$$

$$\Rightarrow \frac{n}{3} - 2 = 2$$

$$\Rightarrow n = 12$$

Hence, the value of n is 12.

107. Show that $2^{4n+4} - 15n - 16,$ where $n \in N$ is divisible by 225.

Ans. :

$$2^{4n+4} - 15n - 16 = 2^{4(n+1)} - 15n - 15 - 1$$

$$= (16)^{n+1} - 15(n+1) - 1$$

$$= (1+15)^{n+1} - 15(n+1) - 1$$

$$= \left[{}^{n+1}C_0 + {}^{n+1}C_1(15) + {}^{n+1}C_2(15)^2 + \ldots.. + {}^{n+1}C_{n+1}(15)^{n+1}\right] - 15(n+1) - 1$$

$$= \left[1 + 15(n+1) + {}^{n+1}C_2(15)^2 + \ldots.. + {}^{n+1}C_{n+1}(15)^{n+1}\right] - 15(n+1) - 1$$

$$= 225\left[{}^{n+1}C_2 + \ldots..{}^{n+1}C_{n+1}(15)^{n-1}\right]$$

= 225 × natural number

108. If in the expansion of $(1+x)^n$, the coefficients of pth and qth term are equal, prove that $p+q=n+2$, where $p \neq q$.

Ans. :

We have,

$(1+x)^n$

Coefficient of pth term $= {}^nC_{p-1}$

Coefficient of qth term $= {}^nC_{q-1}$

It is given that, these coefficients are equal.

${}^nC_{p-1} = {}^nC_{q-1}$

$\Rightarrow p-1 = q-1$ or $p-1+q-1 = n$

$\Rightarrow p-q = 0$ or $p+q = n+2$

$\therefore\ p+q = n+2$

Hence proved.

109. Find the coefficients of a^4 in the product $(1+2a)^4(2-a)^5$ using binomial theorem.

Ans. :

We have,

$(1+2a)^4(2-a)^5$

$= \left[{}^4C_0(2a)^0 + {}^4C_1(2a)^1 + {}^4C_2(2a)^2 + {}^4C_3(2a)^3 + {}^4C_4(2a)^4\right]$

$\times \left[{}^5C_0(2)^5(-a)^0 + {}^5C_1(2)^4(-a)^1 + {}^5C_2(2)^3(-a)^2 + {}^5C_2(2)^2(-a)^3\right.$

$\left. + {}^5C_1(2)^1(-a)^5\right.$

$= \left[1 + 8a + 24a^2 + 32a^3 + 16a^4\right] \times \left[32 - 80a + 80a^2 - 40a^3\right.$

$\left. + 10a^4 - a^5\right]$

Coefficient of $a^4 = 10 - 320 + 1920 - 2560 + 512 = -438$

110. Evaluate the following:

$\left\{a^2 + \sqrt{a^2-1}\right\}^4 + \left\{a^2 - \sqrt{a^2-1}\right\}^4$

Ans. :

$\left\{a^2 + \sqrt{a^2-1}\right\}^4 + \left\{a^2 - \sqrt{a^2-1}\right\}^4$

Let $a^2 = A,\ \sqrt{a^2-1} = B$

$(A+B)^4 + (A-B)^4$

$= B^4 + {}^4C_1AB^3 + {}^4C_2A^2B^2 + {}^4C_3A^3B + A^2 + B^4$

$- {}^4C_1AB^3 + {}^4C_2A^2B^2 - {}^4C_3A^3B + A^4$

$= 2(A^4 + {}^4C_2A^2B^2 + B^4)$

$= 2(A^2 + 6A^2B^2 + B^4)$

$= 2\left(a^8 + 6a^4(a^2-1) + (a^2-1)^2\right)$

$= 2\left[a^8 + 6a^6 - 6a^4 + a^4 + 1 - 2a^2\right]$

$\left\{a^2 + \sqrt{a^2-1}\right\} + \left\{a^2 - \sqrt{a^2-1}\right\}$

$= 2a^8 + 12a^6 - 10a^4 - 4a^4 + 2$

111. If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, prove that its coefficient is $\frac{2n!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}$.

Ans. : Given expression is $\left(x^2 + \frac{1}{x}\right)^{2n}$

General terms, $T_{r+1} = {}^nC_r x^{n-r} y^r$

$= {}^{2n}C_r (x^2)^{2n-r} . \left(\frac{1}{x}\right) = {}^{2n}C_r (x)^{4n-2r} . \frac{1}{x^r}$

$= {}^{2n}C_r (x)^{4n-2r-r} = {}^{2n}C_r (x)^{4n-2r}$

If x^p occurs in $\left(x^2 + \frac{1}{x}\right)^{2n}$

Then $4n - 3r = p \Rightarrow 4n - p$

$\Rightarrow r = \frac{4n-p}{3}$

$\therefore$ Coefficient of $x^p = {}^{2n}C_r = {}^{2n}C_{\frac{4n-p}{3}}$

$= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(2n - \frac{4n-p}{3}\right)!} = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{6n-4n+p}{3}\right)!}$

$= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}$

Hence, the coefficient of $x^p = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}$

112. Find the sixth term of the expansion $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of the third term from the end is 45.

[**Hint:** Binomial coefficient of third term from the end = Binomial coefficient of third term from beginning = ${}^nC_2$.]

Ans. : The given expression is $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)$, since the binomial coefficient of third term from the end = Binomial coefficient of third term from the beginning = ${}^nC_2$

$\therefore {}^nC_2 = 45$

$\Rightarrow \frac{n(n-1)}{2} = 45 \Rightarrow n^2 - n = 90$

$\Rightarrow n^2 - n - 90 = 0 \Rightarrow n^2 - 10n + 9n - 90 = 0$

$\Rightarrow n(n-10) + 9(n-10) \Rightarrow (n-10)(n+9) = 0$

$\Rightarrow n = 10, n = -9 \Rightarrow n = 10, n \neq -9$

So, the given expression becomes $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^{10}$

Sixth term is this expression

$T_6 = T_{5+1} = {}^{10}C_5\left(y^{\frac{1}{2}}\right)^{10-5}\left(x^{\frac{1}{3}}\right)^5 = {}^{10}C_5 y^{\frac{5}{2}}.x^{\frac{5}{3}}$

$= 252\, y^{\frac{5}{2}} x^{\frac{5}{3}}$

Hence, the required term $= 252\, y^{\frac{5}{2}}.x^{\frac{5}{3}}$

----- “Take the attitude of a student, never be too big to ask questions, never know too much to learn something new -----